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An Open Universe from Valley Bounce

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Abstract

It appears difficult to construct a simple model for an open universe based on the one bubble inflationary scenario. The reason is that one needs a large mass to avoid the tunneling via the Hawking Moss solution and a small mass for successful slow-rolling. However, Rubakov and Sibiryakov suggest that the Hawking Moss solution is not a solution for the false vacuum decay process since it does not satisfy the boundary condition. Hence, we have reconsidered the arguments for the defect of the simple polynomial model. We find the valley bounce belonging to a valley line in the functional space represents the decay process instead of the Hawking Moss solution. The point is that the valley bounce gives the appropriate initial condition for the inflation. We show an open inflation model can be constructed within the polynomial form of the potential so that the fluctuations can be reconciled with the observations.

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1 Introduction

Recent observations suggest the matter density of the universe is less than the critical density. Hence, it is desirable to have a model for an open universe, say $\Omega_0 \sim 0.3$. The realization of an open universe is difficult in the ordinary inflationary scenario. This is because if the universe expands enough to solve the horizon problem, the universe becomes almost flat. One attempt to realize an open universe in the inflationary scenario is to consider inside the bubble created by the false vacuum decay [1]. The scenario is as follows. Consider the potential which has two minimum. One is the false vacuum which has non-zero energy and the other is the true vacuum. Initially the field is trapped at the false vacuum. Due to the potential energy, universe expands exponentially and the large fraction of the universe becomes homogeneous. As the false vacuum is unstable, it decays and creates the bubble of the true vacuum. If the decay process is well suppressed, the interior of the bubble is still homogeneous. The decay is described by the $O(4)$ symmetric configuration in the Euclidean spacetime. Then, analytical continuation of this configuration to the Lorentzian spacetime describes the evolution of the bubble which looks from the inside like an open universe. Unfortunately, since the bubble radius cannot be greater than the Hubble radius, the created universe is curvature dominated even if the whole energy of the false vacuum is converted to the energy of the matter inside the bubble [2]. Thus, the second inflation in the bubble is needed. If this second inflation stopped when $\Omega < 1$, our universe becomes homogeneous open universe.

Though the basic idea is simple, the realization of this scenario in a simple model has been recognized difficult [3]. The difficulty is usually explained as follows. Consider the model involving one scalar field. For the polynomial form of the potential like $V(\phi) = m^2\phi^2 - \delta\phi^3 + \lambda\phi^4$, the tunneling should occur at sufficiently large ϕ to ensure that the second inflation gives the appropriate density parameter. Then, the curvature around the barrier which separates the false and the true vacuum is small compared with the Hubble scale which is determined by the energy of the false vacuum. In this case, the field jumps up onto the top of the barrier due to the quantum diffusion. When the field begins to roll down from the top of the barrier, large fluctuations are formed due to the quantum diffusion at the top of the barrier. Then the whole scenario fails. This problem is rather generic. To avoid jumping up, the curvature around the barrier should be large compared with the Hubble scale $V'' > H^2$. On the other hand, to realize the second inflation, the field should roll down slowly, then we need $V'' < H^2$. These two conditions are incompatible.

There are several attempts to overcome this problem. Recently Linde constructs the potential which has sharp peak near the false vacuum [4]. In this potential, the tunneling occurs and at the same time slow-rolling is allowed after tunneling, then the second inflation can be realized. But, it is still unclear what is the physical mechanism for the appearance of the sharp peak in the potential.

We will reconsider this problem from different perspective. The point is the understanding of the tunneling process. In the imaginary-time path-integral formalism, tunneling is described by the solution of the Euclidean field equation. This solution gives the saddle-point of the path-integral. Then this determines the semi-classical exponent of the decay rate $\exp(-S_E(\phi_B))$, where S_E is the Euclidean action. In the case the curvature around the barrier is small compared with the Hubble, the solution is given by the Hawking Moss (HM) solution, which stays at the top of the barrier through the whole Euclidean time [5]. Recently Rubakov and Sibiryakov give the interpretation of this tunneling mode using the constrained instanton method [6, 7]. They show the HM solution does not represent the false vacuum decay if taking into account the analytic continuation to the Lorentzian spacetime, since this solution does not satisfy the boundary condition that the field exists in the false vacuum at the infinite past. However, this does not imply the decay does not occur. One should consider a family of the almost saddle points configurations instead of the true solution of the Euclidean field equation. They show although the decay rate is determined by the HM solution, the structure of the field after tunneling is determined by the other configuration which is one of the almost saddle-point solutions. In this method, one must choose the constraint so that a family of almost saddle-point solutions well covers the region which is expected to dominate the path-integral. One way to realize this is to cover the valley region of the functional space of the action [8]. Along the valley line, the action varies most gently. The action of the configurations on this valley line has a small deviation from that of the HM solution, thus they form a region which is expected to dominate the path-integral. Then it is reasonable to take the configurations on the valley line as a family of the almost saddle points configurations. We will call the configuration on the valley line of the action the valley bounce ϕ_V.

This analysis gives the possibility to overcome the problem. Even if the curvature around the barrier is small compared with the Hubble scale, it implies there is a possibility to occur the tunneling described by the valley bounce. If the field appears sufficiently far from the top, one can avoid the large fluctuations. During the tunneling, fluctuations of the tunneling field are generated. These fluctuations are stretched during the second inflation and observed in the open universe. These should be compatible with the observation. Once this can be confirmed, there is no prevention to construct the one bubble open inflationary model in the simple model with the polynomial form of the potential.

In this paper, we show this is true as long as the tunneling is described by the valley bounce. We clarify the structure of the valley bounce extending the method developed by Aoyama.et.al [8] to the de Sitter spacetime. The fluctuations generated during the tunneling are calculated by defining the fluctuations which are relevant to the observable in the open universe as those orthogonal to the gradient of the action. So far, in the de Sitter spacetime, the analysis of the solution of the Euclidean field equation and the fluctuations around it is restricted to the case one can use the thin-wall approximation [9]. But in general the valley bounce can have a thick-wall profile. Throughout the analysis of the tunneling, we will use the fixed background approximation and the piece-wise quadratic potential [10]. This enables us to solve analytically the valley equation which determines the structure of the valley bounce and the equation of fluctuations around the valley bounce, even if the valley bounce has a thick-wall profile. Then we show an open universe can be constructed from the valley bounce and the fluctuations can be generated with the appropriate properties.

The paper is organized as follows. In the next section we review the formalism to describe the false vacuum decay in the de Sitter spacetime. Then we explain the role of the configurations on the valley of the action i.e. valley bounces. We derive the valley equation which determines the valley bounce. In section 3, we will use the piece-wise quadratic potential and solve the valley equation analytically. In section 4 we calculate the inhomogeneous fluctuations around the valley bounce. These fluctuations give the observable in the open universe. In section 5, we consider the simple model to describe the creation of the open universe. Using the valley bounce, we can construct a model without introducing the fine-tuning of the potential. We calculate the power spectrum of the curvature perturbations generated in this bubble and see these fluctuations are compatible with the observations. In section 6 we summarize the results.

2 Valley method in de-Sitter spacetime

First we review the formalisms which are necessary to describe the false vacuum decay in the de Sitter space. We want to examine the case in which the gravity comes to play a role. Unfortunately, we have not known how to deal with quantum gravity effect yet. So, we study the case in which we can treat gravity at the semi-classical level. That is, we treat the problem within the framework of the field theory in a fixed curved spacetime [6]. The potential relevant to the tunneling is given by

$$V(\phi) = \epsilon + V_T(\phi). \tag{1}$$

We assume ϵ is of the order M_*^4 and $V_T(\phi)$ is of the order M^4. We study the case M is small compared to M_*, $M \ll M_*$. Then the geometry of the spacetime is fixed to the de Sitter spacetime with $H = M_*^2/M_p$,where $M_p^{-2} = 8\pi G/3$. We consider the situation in which the potential $V_T(\phi)$ has the false vacuum at $\phi = \varphi_F$ and the top of the barrier at $\phi = \varphi_T$. Since the background metric is fixed, we can change the origin of the energy freely. We choose $V_T(\phi_F) = 0$. Following, we work in units with $H = 1$.

The decay rate is given by the imaginary part of the path-integral

$$Z = \int [d\phi] \exp\left(-S_E(\phi)\right), \tag{2}$$

where S_E is the Euclidean action relevant to the tunneling. The dominant contribution of this path-integral is given by the configurations which have $O(4)$ symmetry [9]. So, we assume the background metric and the field to have the form

$$\begin{aligned} ds^2 &= d\sigma^2 + a(\sigma)^2 \left(d\rho^2 + \sin^2 \rho d\Omega\right), \\ \phi &= \phi(\sigma), \end{aligned} \tag{3}$$

where $a(\sigma) = \sin\sigma$. Then, the Euclidean action of $\phi(\sigma)$ is given by

$$S_E = 2\pi^2 \int d\sigma \left(a^3 \left(\frac{1}{2}\phi'^2 + V_T(\phi)\right)\right). \tag{4}$$

The saddle-point of this path-integral is determined by the Euclidean field equation $\delta S_E/\delta\phi{=}0$;

$$\phi'' + 3\cot\sigma\,\phi' - V_T'(\phi) = 0. \tag{5}$$

We impose the regularity conditions at the time when $a(\sigma) = 0$ as

$$\phi'(\sigma = 0) = \phi'(\sigma = \pi) = 0. \tag{6}$$

We represent the solution of this equation as $\phi_B(\sigma)$. If the fluctuations around this solution have a negative mode, this gives the imaginary part to the path-integral and this solution contributes to the decay dominantly. The decay rate Γ is evaluated by

$$\Gamma \sim \exp(-S_E(\phi_B)). \tag{7}$$

The equation has two types of the solutions depending on the shape of the potential. If the curvature around the barrier is large compared with the Hubble scale, then the Coleman De Luccia (CD) solution and the Hawking Moss (HM) solution exist [5, 9]. In this case, the decay is described by the CD solution. The analytic continuation of this solution to Lorentzian spacetime describes the bubble of the true vacuum. On the other hand, in the case the curvature around the barrier is small compared with the Hubble scale, only the HM solution exists. This solution is a trivial solution $\phi = \varphi_T$. The meaning of the HM solution is somewhat ambiguous. There are several attempts to interpret this tunneling mode. One way is to use the stochastic approach [11]. Within this approach, it has been demonstrated that the decay rate given by eq.(7) coincides with the probability of jumping from the false vacuum φ_F onto the top of the barrier φ_T due to the quantum fluctuations.

Recently, Rubakov and Sibiryakov give the interpretation of the HM solution using the constrained instanton method [6, 7]. The main idea is to consider a family of the almost saddle-point configurations instead of the true solution of the Euclidean field equation i.e. the HM solution. The motivation comes from the boundary condition. They take the boundary condition that the state of the quantum fluctuations above the classical false vacuum is the conformal vacuum. In this case they show the field should not be constant at $0 < \sigma < \pi$ and the HM solution is excluded by this boundary condition. Then one should seek the other configurations which obey the boundary condition and dominantly contribute to the path-integral. In the functional integral, the saddle-point solution gives the most dominant contribution, but the contribution from a family of almost saddle-points configurations which have almost the same action with that of the saddle-point solution should also be included. To realize this in the functional integral, one introduces the identity $1 = \int d\alpha\delta(\mathcal{C} - \alpha)$ into the path integral for some constraint $\mathcal{C}$. First choose one α. This selects the subspace of the functional space. In this subspace, we can perform the integral of the field using saddle-point method under the constraint. The minimum in this subspace satisfies the equation of motion with constraint instead of the field equation. This minimum corresponds to the almost saddle-point configuration ϕ_α which is slightly deformed from the HM solution. Changing α, these configurations form a trajectory. We can evaluate the path-integral by integrating over α along this trajectory. Since along this trajectory the HM solution gives the minimum action, integrating over α gives the decay rate determined by the HM solution. But the structure of the field after tunneling can be determined by the other configuration on this trajectory ϕ_α. They found the configuration which describes the bubble of the true vacuum if we continue it to the Lorentzian spacetime. Then, they conclude that even in the case only the HM solution exists, the result of the tunneling process can be the bubble of the true vacuum which is described by one of the almost saddle-point configurations.

In this formalism, the validity of the method depends on the choice of the constraint [8, 12]. This is because, in practice, we do the Gaussian integral around the almost saddle-point solutions. To evaluate the path-integral properly we should choose the constraint so that a family of almost saddle-point solutions well covers the region which is expected to dominate the path-integral. Since the action of the configurations on this valley line has a small deviation from that of the saddle-point solution, one way to realize the aim is to cover the valley region of the action [8]. One can identify the configurations on the valley line and make Gaussian integral around these configurations.

In the false vacuum decay process in the flat spacetime, the configurations along the valley line have physical meanings [13]. These configurations actually dominate the path-integral in the presence of the low energy incoming particles. If the incoming particles exist, the path integral which describes the decay is given by $\int[d\phi]\phi^q \exp(-S_E(\phi))$. The effect of the incoming particles deforms the saddle-point. As long as the energy of these incoming particles is low, the deformed configurations belong to the valley. This is because in deforming the configurations from the saddle-point solution, the configurations on the valley line can be obtained most easily compared with the other configurations with the same action. Thus the tunneling with high energy, the configurations on the valley play a crucial role to calculate the decay rate or the cross section. In the de Sitter space time, the choice of the quantum states above a given classical false vacuum will affect the tunneling process considerably [6]. Hence, we think the configurations on the valley line in the de Sitter spacetime may also play an important role, though it is not easy to identify the corresponding quantum state.

Taking into account the above fact, it is desirable to analyze the structure of not only the solution of the Euclidean field equation but also the configurations on the valley. One way to define the configurations on this valley line is to use the valley method developed by Aoyama.et.al [8]. To obtain the intuitive understanding of this method, consider the system of the field ϕ_i. Here i stands for the discretized coordinate label and we take the metric as δ_{ij}. In the valley method the equation which identifies the valley line in the functional space is given by

$$D_{ij}\partial_i S = \lambda\, \partial_i S, \quad D_{ij} = \partial_i\partial_j S, \tag{8}$$

where $\partial_i = \partial/\partial\phi_i$. Since this equation has one parameter λ, this defines a trajectory in the space of ϕ. The parameter λ is one of the eigen value of the matrix D_{ij}. On this trajectory the gradient vector $\partial_i S$ is orthogonal to all the eigenvectors of D_{ij} except for the eigenvector of the eigen value λ. This equation can be rewritten as

$$\partial_i\left(\frac{1}{2}(\partial_j S)^2 - \lambda S\right) = 0. \tag{9}$$

This allows the interpretation of the solution for the equation. It extremizes the norm of the gradient vector $\partial_i S$ under the constraint S =const., where λ plays the role of the Lagrange multiplier. Such solution can be found each hypersurface of constant action, then the solutions of the equation form a line in the functional space. If we take λ as the one with the smallest value, then the gradient vector is minimized. In this case, the action varies most gently along this line. This is a plausible definition of the valley line. We will call the configuration on the valley line of the action the valley bounce ϕ_V and the trajectory they form the valley trajectory.

Following we formulate this method in the de Sitter spacetime. The most convenient way is to use the variational method eq.(9). Define the valley action by

$$S_V = S_E - \frac{1}{2\lambda}\int d\sigma\sqrt{g}\left(\frac{1}{\sqrt{g}}\frac{\delta S_E}{\delta\phi}\right)^2. \tag{10}$$

The valley bounce is obtained by varying this action. The equation which determines the valley bounce $\delta S_V/\delta\phi = 0$ is a fourth order differential equation. We introduce the auxiliary field f to cancel the fourth derivative term [13];

$$S_f = \frac{1}{2\lambda}\int d\sigma\sqrt{g}\left(f - \frac{1}{\sqrt{g}}\frac{\delta S_E}{\delta\phi}\right)^2. \tag{11}$$

Then the valley action becomes

$$S_V + S_f = S_E + \frac{1}{2\lambda}\int d\sigma\sqrt{g}f^2 - \frac{1}{\lambda}\int d\sigma f\frac{\delta S_E}{\delta\phi}. \tag{12}$$

Taking the variation of this action with respect to f and ϕ, we obtain the equations for ϕ and f;

$$\begin{aligned}\frac{1}{\sqrt{g}}\frac{\delta S_E}{\delta\phi} &= f,\\ \int d\sigma'\frac{\delta^2 S_E}{\delta\phi(\sigma)\delta\phi(\sigma')}f(\sigma) &= \lambda\sqrt{g}f(\sigma).\end{aligned} \tag{13}$$

Using $a(\sigma) = \sin\sigma$, the valley equation which determines the structure of the valley bounce is given by

$$\begin{aligned}\phi'' + 3\cot\sigma\,\phi' - V_T'(\phi) &= -f,\\ f'' + 3\cot\sigma\, f' - V_T''(\phi)f &= -\lambda f.\end{aligned} \tag{14}$$

To evaluate the path-integral using the valley bounces, we parameterize the valley trajectory by α; $\phi_V(\alpha) = \phi_\alpha$. The valley bounces on this trajectory play a role as a family of the almost saddle-point solutions. We restrict our attention to the $O(4)$ symmetric configurations. We expand the Euclidean action around the valley bounce $\phi_V(\alpha) = \phi_\alpha$;

$$S_E(\phi) = S_E(\phi_\alpha) + \int d\sigma\sqrt{g}\frac{1}{\sqrt{g}}\left.\frac{\delta S_E}{\delta\phi}\right|_{\phi_\alpha}\delta\phi(\sigma) + \frac{1}{2}\int\int d\sigma d\sigma'\left.\frac{\delta^2 S_E}{\delta\phi(\sigma)\delta\phi(\sigma')}\right|_{\phi_\alpha}\delta\phi(\sigma)\delta\phi(\sigma'). \tag{15}$$

Since the valley bounce does not obey the field equation, then the first order derivative term does not vanish. This can be avoided by constraining the space of the fluctuations to that orthogonal to the gradient of the action;

$$G(\phi_\alpha) = \int d^4x\sqrt{g}\,\left(\frac{1}{\sqrt{g}}\frac{\delta S_E}{\delta\phi}\bigg|_{\phi_\alpha}\delta\phi\right) = 0. \tag{16}$$

The path-integral can be rewritten by using Faddev-Popov method. Inserting the unity

$$1 = \int d\alpha\ \delta\left(G(\phi_\alpha)\right)\det\left(\frac{\delta G(\phi_\alpha)}{\delta\alpha}\right), \tag{17}$$

to the path-integral. Then, the path-integral becomes

$$Z_0 = \int d\alpha' \exp(-S_E(\phi_\alpha))\int[d\delta\phi]\delta\left(G(\phi_\alpha)\right)\exp\left(-\frac{1}{2}\int\int d\sigma d\sigma'\ \frac{\delta^2 S_E}{\delta\phi(\sigma)\delta\phi(\sigma')}\bigg|_{\phi_\alpha}\delta\phi(\sigma)\delta\phi(\sigma')\right), \tag{18}$$

where

$$d\alpha' = d\alpha\left|\int d\sigma\sqrt{g}\,\left(\frac{d\phi_\alpha}{d\alpha}\frac{1}{\sqrt{g}}\frac{\delta S_E}{\delta\phi}\right)\right|. \tag{19}$$

The Jacobian factor arises because the trajectory is not necessarily orthogonal to the chosen subspace. The fluctuations can be expanded by the eigenmodes $g_{\alpha,n}(\sigma)$ with the eigenvalue $\rho_{\alpha,n}$ of the operator $(\delta S_E^2/\delta\phi\delta\phi)_{\phi_\alpha}$;

$$g''_{\alpha,n} + 3\cot\sigma\ g'_{\alpha,n} - V''_T(\phi)g_{\alpha,n} = -\rho_{\alpha,n}g_{\alpha,n}. \tag{20}$$

Since λ is one of the $\rho_{\alpha,n}$ with the smallest value, the gradient of the action $f(\sigma)$ is orthogonal to the other eigenmodes with $\rho_{\alpha,n} \neq \lambda$. Thus, the fluctuations orthogonal to the gradient of the action are expanded as

$$\delta\phi = {\sum_n}' X_n g_{\alpha,n}, \tag{21}$$

where $'$ stands for the sum without n which gives $\rho_{\alpha,n} = \lambda$. Using this expansion, the integration of $\delta\phi$ becomes

$$\begin{aligned}
&\int[d\delta\phi]\delta\left(G(\phi_\alpha)\right)\exp\left(-\frac{1}{2}\int\int d\sigma d\sigma'\ \frac{\delta^2 S_E}{\delta\phi(\sigma)\delta\phi(\sigma')}\bigg|_{\phi_\alpha}\delta\phi(\sigma)\delta\phi(\sigma')\right)\\
=&\int{\prod_n}' dX_n\exp\left(-\frac{1}{2}{\sum_n}'\rho_{\alpha,n}X_n^2\right)\\
=&\frac{1}{\sqrt{\prod'\rho_{\alpha,n}}},
\end{aligned} \tag{22}$$

where $\prod'\rho_{\alpha,n} = (\prod_n\rho_{\alpha,n})/\lambda$. Thus, the path-integral is evaluated as

$$Z_0 \sim \int d\alpha'\frac{1}{\sqrt{\prod'\rho_{\alpha,n}}}\exp(-S_E(\phi_\alpha)). \tag{23}$$

$S_E(\phi_\alpha)$ has a minimum at the solution of the Euclidean field equation $\phi_\alpha = \phi_B$. However in the case the solution is the Hawking-Moss solution, this solution does not contribute to the path-integral due to the boundary condition. Instead of this solution, one of the valley bounces ϕ_V plays a role. To ensure that this solution gives the imaginary part to the path-integral, the fluctuations around the valley bounce should have one negative eigenvalue ρ_-.

In the next section, we solve the valley equation and clarify the structure of the valley bounce. Solving the valley equation is the eigenvalue problem of the two variables, it is desirable to solve the equation analytically to confirm the existence of the solutions. The valley bounce can have a thick-wall profile. In the flat spacetime, there exists the attempt to treat thick-wall solutions analytically by constructing the piece-wise quadratic potentials [10]. In the next section, we extend this attempt to the de Sitter spacetime and solve not only the Euclidean field equation but also the valley equation analytically.

3 Valley bounces

3.1 The construction of the Valley bounces

To solve the valley equation developed in the last section, we construct the piece-wise quadratic potential. We connect two parabola. In this potential the true vacuum is absent. But this is not essential in calculations. In fact we have solved numerically the valley equation for several potentials and found this model is sufficient to discuss the generic feature of the valley bounce.

The potential which we study is

$$V_T(\phi) = \begin{cases} \frac{1}{2}m_F^2(\phi-\varphi_F)^2, & -\infty < \phi < 0, \\ \\ -\frac{1}{2}m_T^2(\phi-\varphi_T)^2+\eta, & 0 \le \phi < \infty, \end{cases} \tag{24}$$

where η is of the order M^4. We require that the potential and its derivative are connected smoothly at the connection point $\phi = 0$. From this condition, we obtain

$$\begin{aligned} \varphi_T &= -\frac{m_F^2}{m_T^2}\varphi_F, \\ \varphi_F &= -\sqrt{\frac{2m_T^2\,\eta}{m_F^2(m_F^2+m_T^2)}}. \end{aligned} \tag{25}$$

φ_T and φ_F have a mass scale of the order M^2/m_T. Thus, we rescale the field as $\phi \to (M^2/m_T)\phi$ and $f \to (M^2/m_T)f$.

First we solve the Euclidean field equation;

$$\phi'' + 3\cot\sigma\,\phi' - V_T'(\phi) = 0. \tag{26}$$

If one puts $z = -\cos\sigma$ and $Y(z) = \sqrt{1-z^2}(\phi - \varphi_i)$, this equation reduces to the associated Legendre differential equation

$$(1-z^2)\frac{d^2Y}{dz^2} - 2z\frac{dY}{dz} + \left[\nu_i(\nu_i+1) - \frac{\mu^2}{1-z^2}\right] Y = 0, \tag{27}$$

where $\mu = 1$ and ν_i is given by

$$\nu_T = \sqrt{\frac{9}{4} + m_T^2} - \frac{1}{2}, \quad \nu_F = \sqrt{\frac{9}{4} - m_F^2} - \frac{1}{2}. \tag{28}$$

Here, $i = T$ for $-\infty < \phi < 0$ and $i = F$ for $0 \leq \phi < \infty$. The independent solutions of this equation are given by the associated Legendere function of the first and second kinds, $P^1_{\nu_i}(z)$ and $Q^1_{\nu_i}(z)$. $P^1_\nu(z)$ is regular at $z \to 1$. Since these functions behave at $z \to -1$ as

$$\begin{aligned} P^1_\nu(z) &\rightarrow -2^{1/2}\sin(\pi\nu)\pi^{-1}(1+z)^{-1/2}, \\ Q^1_\nu(z) &\rightarrow -2^{-1/2}\cos(\pi\nu)(1+z)^{-1/2}, \end{aligned} \tag{29}$$

the combination of these solutions

$$B^\mu_\nu(z) = P^\mu_\nu(z) + \left(-\frac{2}{\pi}\tan(\pi\nu)\right) Q^\mu_\nu(z) \tag{30}$$

is regular at $z \to -1$. Then the solution which satisfies the boundary condition is given by

$$\phi_B = \begin{cases} \varphi_F + \dfrac{1}{\sqrt{1-z^2}} A_F\, B^1_{\nu_F}(z), & -1 \leq z < z_0, \\ \\ \varphi_T + \dfrac{1}{\sqrt{1-z^2}} A_T\, P^1_{\nu_T}(z), & z_0 \leq z \leq 1, \end{cases} \tag{31}$$

where $\phi_B(z_0) = 0$. Since the potential is constructed to be smooth to its first derivative, we demand ϕ_B and its first derivative must be continuous at $z = z_0$. Then the coefficients A_i are determined in terms of z_0;

$$A_F(z_0) = -\frac{\varphi_F\sqrt{1-z_0^2}}{B^1_{\nu_F}(z_0)}, \quad A_T(z_0) = -\frac{\varphi_T\sqrt{1-z_0^2}}{P^1_{\nu_T}(z_0)}. \tag{32}$$

The junction time z_0 is determined by $\phi_B(z_0) = 0$;

$$\varphi_F P^1_{\nu_T}(z_0) B^2_{\nu_F}(z_0) - \varphi_T P^2_{\nu_T}(z_0) B^1_{\nu_F}(z_0) = 0. \tag{33}$$

If this algebraic equation for z_0 has a solution, this gives the CD solution. The condition for the existence of the solution restricts the parameter m_i. We see this condition is approximately given by $m_T^2 > 4$.

Next we shall solve the valley equation. Equation for f is given by

$$f'' + 3\cot\sigma\, f' - V_T''(\phi) f = -\lambda f. \tag{34}$$

The regularity conditions are the same with that of ϕ. Then, the general solution which satisfies the boundary conditions is given by

$$f = \begin{cases} \dfrac{1}{\sqrt{1-z^2}} G_F \, B^1_{\nu_{F\lambda}}(z), & -1 \le z < z_\lambda, \\ \\ \dfrac{1}{\sqrt{1-z^2}} G_T \, P^1_{\nu_{T\lambda}}(z), & z_\lambda \le z \le 1, \end{cases} \tag{35}$$

where

$$\nu_{T\lambda} = \sqrt{\frac{9}{4} + (m_T^2 + \lambda)} - \frac{1}{2}, \quad \nu_{F\lambda} = \sqrt{\frac{9}{4} - (m_F^2 - \lambda)} - \frac{1}{2}. \tag{36}$$

From the junction conditions, we obtain

$$G_F = \frac{P^1_{\nu_{T\lambda}}(z_\lambda)}{B^1_{\nu_{F\lambda}}(z_\lambda)} G_T, \quad G_F = \frac{P^2_{\nu_{T\lambda}}(z_\lambda)}{B^2_{\nu_{F\lambda}}(z_\lambda)} G_T. \tag{37}$$

This equation has solutions only if z_λ satisfies the following equation

$$P^1_{\nu_{T\lambda}}(z_\lambda) B^2_{\nu_{F\lambda}}(z_\lambda) - P^2_{\nu_{T\lambda}}(z_\lambda) B^1_{\nu_{F\lambda}}(z_\lambda) = 0. \tag{38}$$

This equation determines the junction time z_λ. Next we solve the equation for ϕ;

$$\phi'' + 3 \cot \sigma \, \phi' - V_T'(\phi) = -f. \tag{39}$$

In this equation, f acts as the source. We can see that the special solution is given by

$$\phi - \varphi_i = \frac{f}{\lambda}. \tag{40}$$

Then the solution which satisfies the boundary condition is given by

$$\phi_V = \begin{cases} \varphi_F + \dfrac{1}{\sqrt{1-z^2}} A_F \, B^1_{\nu_F}(z) + \dfrac{1}{\lambda\sqrt{1-z^2}} G_F \, B^1_{\nu_{F\lambda}}(z), & -1 \le z < z_\lambda, \\ \\ \varphi_T + \dfrac{1}{\sqrt{1-z^2}} A_T \, P^1_{\nu_T}(z) + \dfrac{1}{\lambda\sqrt{1-z^2}} G_T \, P^1_{\nu_{T\lambda}}(z), & z_\lambda \le z \le 1. \end{cases} \tag{41}$$

From the junction conditions, we obtain the coefficients

$$\begin{aligned} A_F &= \sqrt{1-z_\lambda^2} \frac{P^2_{\nu_T}(z_\lambda)(\varphi_F - \varphi_T)}{P^1_{\nu_T}(z_\lambda) B^2_{\nu_F}(z_\lambda) - P^2_{\nu_T}(z_\lambda) B^1_{\nu_F}(z_\lambda)}, \\ G_F &= -\lambda \sqrt{1-z_\lambda^2} \frac{\varphi_F P^1_{\nu_T}(z_\lambda) B^2_{\nu_F}(z_\lambda) - \varphi_T P^2_{\nu_T}(z_\lambda) B^1_{\nu_F}(z_\lambda)}{B^1_{\nu_{F\lambda}}(z_\lambda)(P^1_{\nu_T}(z_\lambda) B^2_{\nu_F}(z_\lambda) - P^2_{\nu_T}(z_\lambda) B^1_{\nu_F}(z_\lambda))}, \\ A_T &= \frac{B^2_{\nu_F}(z_\lambda)}{P^2_{\nu_T}(z_\lambda)} A_F, \qquad G_T = \frac{B^2_{\nu_{F\lambda}}(z_\lambda)}{P^2_{\nu_{T\lambda}}(z_\lambda)} G_F. \end{aligned} \tag{42}$$

Note that in this model the deformation of the configurations is essentially determined by z_λ. If $z_\lambda = z_0$, G_i becomes 0, so $\phi_V = \phi_B$ as expected.

To ensure that the valley bounce plays a role instead of the true saddle point solution, we must examine the fluctuations around the valley bounce have one negative mode and give the imaginary part to the path-integral. The equation which determines the eigenmodes is given by

$$g_n'' + 3\cot\sigma\, g_n' - V_T''(\phi)g_n = -\rho_n g_n. \tag{43}$$

Then, the eigenvalue equation which determines the eigenvalue ρ_n of these eigenmodes becomes

$$P^1_{\nu_{T\rho_n}}(z_\lambda)B^2_{\nu_{F\rho_n}}(z_\lambda) - P^2_{\nu_{T\rho_n}}(z_\lambda)B^1_{\nu_{F\rho_n}}(z_\lambda) = 0. \tag{44}$$

Note that, λ is one of the solutions ρ_n.

We should treat separately the case in which the valley bounce exists around the top of the barrier and passes through only one parabola. We put the solution for f as

$$f = \sum_{n=0}^{\infty} b_n \cos n\sigma, \tag{45}$$

then the equation for f is rewritten as [14]

$$\sum_{n=0}^{\infty}\Big([(n-1)(n+2) - m_T^2 - \lambda]b_{n-1} - [(n+1)(n-2) - m_T^2 - \lambda]b_{n+1}\Big)\sin n\sigma = 0. \tag{46}$$

Thus, b_n converges only when

$$\lambda = -m_T^2 + n(n+3). \tag{47}$$

We put the solution for ϕ as

$$\phi - \varphi_T = \sum_{n=0}^{\infty} a_n \cos n\sigma, \tag{48}$$

then the solution for ϕ is given by

$$a_n = \frac{1}{\lambda}c_n. \tag{49}$$

The eigenvalue of the eigenmode at the valley bounce is given by

$$\rho_n = -m_T^2 + n(n+3). \tag{50}$$

3.2 The structure of the valley

Using the analytic solution of ϕ and f, we show the structure of the valley. Remember that we have rescaled the field as $\phi \to (M^2/m_T)\phi$ and $f \to (M^2/m_T)f$. Following, for completeness, we consider the two types of the potential; (1)$m_T^2 > 4$ and (2)$m_T^2 < 4$.

(1)$m_T^2 > 4$

In this case there exist two solutions in the Euclidean field equation; the CD solution and the HM solution. For example, we take $m_T^2 = 7$, $m_F^2 = 2.2$ and $\eta = 0.6M^4$ (Fig.1). The behaviors of the CD solution $\phi(\sigma)$, the eigenmode with the negative eigenvalue $g_-(\sigma)$ and the scale factor $a(\sigma)$ are shown in Fig.2. The CD solution has one negative eigenvalue $\rho_{CD,-} = -4.7$ and the smallest positive eigenvalue is $\rho_{CD,+} = 3$. Since the CD solution gives the saddle-point of the path integral, we analyze the valley trajectory which contains the CD solution. At the CD solution on the valley trajectory, $f = 0$. The valley bounce near the CD solution is obtained by deforming the CD solution; $\phi_V = \phi_{CD} + \triangle\phi_V$. The deformation $\triangle\phi_V$ is due to the source term $f \neq 0$ in the equation for ϕ. The equation for f is almost the same with that for the eigenmode g at the CD solution. Thus, λ is given by $\lambda = \rho_{CD} + \triangle\lambda$. To ensure the action varies most gently along the valley trajectory, ρ_{CD} should be the eigenvalue of the smallest value. In this case, ρ_{CD} has one negative eigenvalue, so we take λ at the CD solution as $\lambda(\phi_{CD}) = \rho_{CD,-}$ or $\lambda(\phi_{CD}) = \rho_{CD,+}$.

First examine the valley trajectory associated with the negative eigenvalue ($\lambda(\phi_{CD}) = \rho_{CD,-}$). The valley bounce obtained from the analytic results developed in the previous section is shown in the lower-panel of Fig.3. Since $\rho_{CD,-}$ is the lowest eigenvalue, f does not have a node. In the equation for ϕ, f acts as the force. So, the valley bounce in this trajectory is obtained by deforming the CD solution adding a one-direction force f. If $f > 0$, the valley bounce has a structure of the small bubble and if $f < 0$ it has a structure of the large bubble [13]. We plot the action along this trajectory in lower-panel of Fig.4. The CD solution gives the maximum of the action.

Next consider the valley trajectory associated with the smallest positive eigenvalue ($\lambda(\phi_{CD}) = \rho_{CD,+}$). We show the valley bounce in this trajectory in the upper panel of Fig.3. Since $\rho_{CD,+}$ is the next to the lowest eigenvalue, f has one node. In the equation for ϕ, f acts as the mass term of ϕ. So, in this trajectory, the valley bounce is obtained by modifying the mass of the field ϕ. It is known that the CD solution is smoothly connected to the HM solution if one decreases the mass around the top of the barrier [15]. From this fact, it is expected that this trajectory connects the CD solution and the HM solution. The action along this Valley trajectory is shown in the upper-panel of Fig.4. Since the degeneracy occurs in λ, we take the horizontal coordinate as the 'norm' of the solution $|\Phi| = \sqrt{2\pi^2 \int a^3(\phi - \varphi_T)^2}$ [13]. From this, we see the CD solution is the minimum and the HM solution is the maximum of the action and these solutions are smoothly connected on this trajectory as expected.

(2)$m_T^2 < 4$.

In this case only the HM solution exists. For example we take $m_T^2 = 2$, $m_F^2 = 0.5$ and $\eta = 0.1M^4$ (Fig.5). The HM solution has one negative eigenvalue $\rho_{HM,-} = -2$ and the smallest positive eigenvalue is given by $\rho_{HM,+} = 2$. The generic feature of the valley bounce is understood by the simple analysis of the case in which the valley bounce exists only in one parabola. First consider the valley trajectory associated with the negative eigenvalue. The solution of the valley equation is essentially has a form $f = \lambda(\phi - \varphi_T) = \text{const}$. This solution does not represent the tunneling, so we seek the trajectory associated with the smallest positive eigenvalue $\lambda(\phi_{HM}) = \rho_{HM,+}$. The solution of the valley equation is given by $\phi - \phi_T \propto \cos\sigma$ and $f = \lambda(\phi - \phi_T)$ (Fig.7). In this trajectory, the HM solution gives the minimum of the action (Fig.6). The action grows as the variation of the field becomes large, but this increase is relatively small.

Although the HM solution gives the dominant contribution to the path-integral, this solution does not satisfy the boundary condition for the false vacuum decay as shown by Rubakov and Sibiryakov [6]. Making Analytic continuation to the Lorentzian spacetime at $\sigma = 0 (z = -1)$, the field moves according to the field equation. If the field reaches φ_F, this solution represents the false vacuum decay. The behavior of the field in this Lorentzian spacetime is determined by the initial position of the field. This is determined by the behavior of the field at $\sigma = 0$ in the Euclidean region. Provided that its initial position is different from φ_T, this boundary condition can be satisfied. From this fact, the HM solution does not satisfy the boundary condition. On the other hand the valley bounce does satisfy the boundary condition. Furthermore the fluctuations around the valley bounce should have one negative mode to ensure that the valley bounce plays a role instead of the HM solution. The valley bounce has a lowest eigenvalue $\rho_{V,-} < \lambda(\phi_V)$. We find this is negative on this trajectory. Since this is the unique negative eigenvalue, the gaussian integration of the fluctuations around this valley bounce gives the imaginary part to the path integral. Then, this almost saddle-point solution contributes to the false vacuum decay and describes the creation of the bubble of the true vacuum.

4 Fluctuations around valley bounce

So far we restrict our attention to the $O(4)$ symmetric configurations. In this section we consider the fluctuations which do not have $O(4)$ symmetric configurations. These fluctuations become the spatially inhomogeneous fluctuations in the open universe. We represent the valley bounce which plays a role for the saddle-point solution ϕ_0. The structure of the bubble created by the decay is described by this configuration ϕ_0. The path-integral including the inhomogeneous fluctuations is given by

$$Z = Z_0 \int [d\delta\phi] \delta\left(G(\phi_0)\right) \exp\left(-\frac{1}{2} \int\int dx^4 dx'^4 \left. \frac{\delta^2 S_E}{\delta\phi(x)\delta\phi(x')} \right|_{\phi_0} \delta\phi(x)\delta\phi(x') \right), \tag{51}$$

Here Z_0 is the part obtained from the $O(4)$ symmetric configurations. In the de Sitter spacetime, fluctuations are expanded by scalar harmonics

$$\delta\phi(\sigma, \rho, \Omega) = \int dp\, S_p(\sigma) Y_{plm}(\rho, \Omega). \tag{52}$$

The harmonics obeys the orthogonal relation between different p. Now the gradient vector is given by $f(\sigma)$. This corresponds to the mode of $p = i$. Then, the inhomogeneous fluctuations which depend on ρ, Ω ($p \neq i$) are orthogonal to the gradient of the action $f(\sigma)$ automatically. We must calculate the physical observable like two-point correlation function in the bubble described by ϕ_0. For example, consider the variance of the scalar field $\langle \phi^2 \rangle - \langle \phi \rangle^2$, where $\langle\ \rangle$ is average over ρ, Ω. Analytically continuing to the Lorentzian spacetime, this corresponds to the average over the space in open universe. Using the relation

$$\langle \phi_0 \rangle = \phi_0, \tag{53}$$

we can show

$$\langle\phi^2\rangle - \langle\phi\rangle^2 = \langle\delta\phi^2\rangle - \langle\delta\phi\rangle^2. \tag{54}$$

So the observable in the open universe can be evaluated from these fluctuations.

Fluctuations around the valley bounce ϕ_0 obey the field equation

$$\int dx'^4 \left.\frac{\delta^2 S_E}{\delta\phi(x)\delta\phi(x')}\right|_{\phi_0} \delta\phi(x') = 0. \tag{55}$$

Following, we solve this equation. Though the physical situation is different, the calculation is the same with that given by previous works [16, 17, 18, 19]. We will follow their calculations. In calculating the fluctuations, we must specify the initial state of the fluctuations. To do this, first consider the analytic continuation to the Lorentzian de Sitter spacetime. We obtain the coordinate systems in the Lorentzian de Sitter space from the Euclidean metric

$$ds^2 = d\sigma^2 + a(\sigma)^2\left(d\rho^2 + \sin^2\rho d\Omega^2\right), \tag{56}$$

by the analytic continuation

$$\tau = i(\rho - \pi/2), \;\; \sigma = \sigma. \tag{57}$$

The resulting metric is given by

$$ds^2 = d\sigma^2 + a(\sigma)^2\left(-d\tau^2 + \cosh^2\tau d\Omega^2)\right). \tag{58}$$

We take the nucleation surface of the bubble at $\tau = 0$ surface. Expanding the fluctuations as

$$\delta\phi(\sigma,\tau,\Omega) = \int dp\, S_p(\sigma) Y_{plm}(\tau,\Omega), \tag{59}$$

we obtain the equation of the fluctuations

$$\left(\frac{\partial^2}{\partial\tau^2} + 2\tanh\tau\frac{\partial}{\partial\tau} + \frac{l(l+1)}{\cosh^2\tau}\right) Y_{plm}(\tau,\Omega) = -(1+p^2)Y_{plm}(\tau,\Omega), \tag{60}$$

$$S_p''(\sigma) + 3\cot\sigma S_p'(\sigma) + \left(\frac{1+p^2}{\sin^2\sigma} - V_T''(\phi_0)\right) S_p(\sigma) = 0, \tag{61}$$

where

$$V_T''(\phi_0) = \begin{cases} m_F^2, & 0 \le \sigma < \sigma_\lambda, \\ -m_T^2, & \sigma_\lambda \le \sigma \le \pi, \end{cases}$$

and $z_\lambda = -\cos\sigma_\lambda$. Here λ is determined by ϕ_0. Since the temporal coordinate τ is included in the harmonics Y_{plm}, the choice of the solution Y_{plm} specifies the vacuum. This choice is related to the initial quantum state of the fluctuations. We will take this initial state as Bunch-Davis vacuum. The equation for $S_p(\sigma)$ can be rewritten as

$$\left(-\frac{d^2}{du^2} + U(u)\right)\left(\frac{S_p(u)}{a(u)}\right) = p^2\left(\frac{S_p(u)}{a(u)}\right), \tag{62}$$

where

$$a(u) = (\cosh u)^{-1}, \quad U(u) = \frac{V''(\phi_0) - 2}{\cosh^2 u}, \quad \tanh u = -\cos\sigma = z. \tag{63}$$

Since $U(u) \to 0$ as $u \to \pm\infty$, the modes are continuous for $p^2 > 0$. For $u_\lambda < u$, the potential has a valley, some discrete modes exist for $p^2 < 0$.

First consider the continuous modes. Positive frequency mode should satisfy the Klein-Gordon normalization

$$-i \int dz \quad \left[\cosh^2\tau \, d\Omega \left(\delta\phi^+_{plm} \ (\partial_\tau \delta\phi^{+*}_{p'l'm'}) - \ (\partial_\tau \delta\phi^+_{plm}) \ \delta\phi^{+*}_{p'l'm'}\right)\right]_{\tau=0} = \delta(p-p')\delta_{ll'}\delta_{mm'}. \tag{64}$$

For simplicity we consider s-wave. The normalized positive frequency mode function of the Bunch-Davis vacuum is given by

$$\delta\phi^+_{\pm p}(\sigma,\tau) = S_{\pm p}(\sigma) Q_p(\tau), \;\; Q_p(\tau) = \frac{e^{\pi p/2} e^{-ip\tau} - e^{-\pi p/2} e^{ip\tau}}{\sqrt{2\sinh \pi p} \quad \cosh\tau}, \tag{65}$$

where S_p is normalized as

$$\int_{-1}^{1} dz \quad S_p(z) S^*_{p'}(z) = \frac{1}{8\pi |p|} \delta(p-p'). \tag{66}$$

Using this mode function, the fluctuations can be expanded as

$$\delta\phi = \int_0^\infty dp \quad \left[(\delta\phi^+_p \ \hat{\boldsymbol{a}}_p + \delta\phi^+_{-p} \ \hat{\boldsymbol{a}}_{-p}) + (h.c)\right], \tag{67}$$

where $\hat{\boldsymbol{a}}_p$ annihilates the Bunch-Davis vacuum.

We take the initial fluctuations $S_p(z)$ at $z = -1$ as the Klein-Goldon normalized mode $F^F_p(z)$ on $-1 \le z \le 1$, then we evolve this mode using the field equation to $z = 1$. The resulting mode function is

$$\tilde{S}_p(z) = \begin{cases} F^F_p(z), & -1 \le z < z_\lambda \\ \\ \alpha_p F^T_p(z) + \beta_p F^T_{-p}(z), & z_\lambda \le z \le 1, \end{cases} \tag{68}$$

where

$$F^i_p(z) = \frac{1}{\sqrt{1-z^2}} \frac{1}{4\pi\sqrt{|p|}} \left(a^i_+ \Gamma(1-ip) P^{ip}_{\nu_i}(z) - a^i_- \Gamma(1+ip) P^{-ip}_{\nu_i}(z)\right), \tag{69}$$

and

$$\begin{aligned} a^i_+ &= \sqrt{\frac{1+\sqrt{1-|C^i_2|^2/C^{i2}_1}}{2}}, \quad a^i_- = \left(\frac{C^i_2}{|C^i_2|}\right)\sqrt{\frac{1-\sqrt{1-|C^i_2|^2/C^{i2}_1}}{2}}, \\ C^i_1(p) &= 2\pi\left(1+\frac{\sin^2\pi\nu_i}{\sinh^2\pi p}\right), \quad C^i_2(p) = -2\pi^2 \frac{\Gamma[1-ip]}{\Gamma[1+ip]} \frac{\sin\pi\nu_i}{\sinh^2\pi p} \frac{1}{\Gamma[-ip-\nu_i]\Gamma[1-ip+\nu_i]}. \end{aligned}$$

Here, α_p and β_p are determined by the junction conditions at z_λ

$$
\begin{aligned}
\alpha_p(z_\lambda) &= \left.\frac{F_p^F d_z F_{-p}^T - F_{-p}^T d_z F_p^F}{F_p^T d_z F_{-p}^T - F_{-p}^T d_z F_p^T}\right|_{z_\lambda}, \\
\beta_p(z_\lambda) &= \left.\frac{-F_p^F d_z F_p^T + F_p^T d_z F_p^F}{F_p^T d_z F_{-p}^T - F_{-p}^T d_z F_p^T}\right|_{z_\lambda},
\end{aligned} \tag{70}
$$

where $d_z = d/dz$. $\tilde{S}_p(z)$ is not be normalized on $-1 \leq z \leq 1$. The normalized mode function is given by

$$
S_p(z) = \begin{cases} b_+ F_p^F(z) - b_- F_{-p}^F(z), & -1 \leq z < z_\lambda \\ (b_+\alpha_p - b_-\beta_{-p})F_p^T(z) + (b_+\beta_p - b_-\alpha_{-p})F_{-p}^T(z), & z_\lambda \leq z \leq 1, \end{cases} \tag{71}
$$

where

$$
b_+ = \sqrt{\frac{D_1}{D_1^2 - |D_2|^2}}\sqrt{\frac{1+\sqrt{1-|D_2|^2/D_1}}{2}}, \quad b_- = \left(\frac{D_2}{|D_2|}\right)\sqrt{\frac{D_1}{D_1^2 - |D_2|^2}}\sqrt{\frac{1-\sqrt{1-|D_2|^2/D_1}}{2}}, \tag{72}
$$

and

$$
\begin{aligned}
D_1(p) &= \frac{1}{2}(|\tilde{\alpha}_p|^2 + |\tilde{\beta}_p|^2 + 1), \quad D_2(p) = \tilde{\alpha}_p \tilde{\beta}_p + \frac{C_2^F}{2C_1^F}, \\
\tilde{\alpha}_p &= \alpha_p a_+^T - \beta_p a_-^{T*}, \quad \tilde{\beta}_p = \beta_p a_+^T - \alpha_p a_-^T.
\end{aligned} \tag{73}
$$

The fluctuations propagate into inside the bubble across the light cone $\sigma = \pi (z = 1)$. Since the coordinate system (σ, τ, Ω) is singular at this point, we make analytic continuation by

$$
r = \tau + i\frac{\pi}{2}, \quad t = i(\sigma - \pi). \tag{74}
$$

The resulting metric is given by

$$
ds^2 = -dt^2 + b(t)^2\left(dr^2 + \sinh^2 r d\Omega^2\right), \tag{75}
$$

where $b(t) = \sinh t$. Since during the second inflation in the bubble the fluctuations exponentially expand, the shortwavelength modes are relevant. The matching condition across the lightcone in the Minkowski limit is given by

$$
\begin{aligned}
F_p^T(\sigma)Q_p(\tau) &\rightarrow \frac{-i}{2\sqrt{2p}}\frac{1}{\sqrt{2\sinh \pi p}} R_p(r)(a_+^T e^{\pi p/2} T_p(\eta) - a_-^T e^{-\pi p/2} T_{-p}(\eta)), \\
F_{-p}^T(\sigma)Q_p(\tau) &\rightarrow \frac{-i}{2\sqrt{2p}}\frac{1}{\sqrt{2\sinh \pi p}} R_p(r)(a_+^T e^{-\pi p/2} T_{-p}(\eta) - a_-^{T*} e^{\pi p/2} T_p(\eta)),
\end{aligned} \tag{76}
$$

where

$$
\begin{aligned}
T_p(\eta) &= e^{-ip\eta - \eta}, \ e^\eta = \tanh(t/2), \\
R_p(r) &= \frac{1}{\sqrt{2}\pi}\frac{\sin p\, r}{\sinh r}.
\end{aligned} \tag{77}
$$

Note that $R_p(r)$ is the normalized scalar harmonics $R_p(r) = Y_{p00}(r)$, where

$$Y_{plm}(r,\Omega) = \frac{p\Gamma[ip+l+1]}{\Gamma[ip+1]} \frac{P^{-l-1/2}_{ip-1/2}(\cosh r)}{\sqrt{\sinh r}} Y_{lm}(\Omega), \tag{78}$$

and Y_{lm} is the usual spherical harmonics. Then, the extension to the general modes with $l \neq 0, m \neq 0$ is straightforwardly given by replacing $R_p(r)$ to $Y_{plm}(r,\Omega)$. We obtain the fluctuations inside the bubble

$$\begin{aligned} \delta\phi &= -i\sum_{lm}\int_0^\infty dp \frac{1}{2\sqrt{2p}} Y_{plm}(r,\Omega)\frac{1}{\sqrt{2\sinh \pi p}} \\ &\times \Big[\Big((e^{\pi p/2} g_1(p,\lambda)T_p(\eta) + e^{-\pi p/2} g_2(p,\lambda)T_{-p}(\eta))\hat{\boldsymbol{a}}_p \\ &+ (e^{-\pi p/2} g_1^*(p,\lambda)T_{-p}(\eta) + e^{\pi p/2} g_2^*(p,\lambda)T_p(\eta))\hat{\boldsymbol{a}}_{-p}\Big) + (h.c.)\Big], \end{aligned} \tag{79}$$

where

$$\begin{aligned} g_1(p,\lambda) &= a_+^T(b_+\alpha_p(z_\lambda) - b_-\beta_{-p}(z_\lambda)) - a_-^{T*}(b_+\beta_p(z_\lambda) - b_-\alpha_{-p}(z_\lambda)) \\ g_2(p,\lambda) &= a_+^T(b_+\beta_p(z_\lambda) - b_-\alpha_{-p}(z_\lambda)) - a_-^T(b_+\alpha_p(z_\lambda) - b_-\beta_{-p}(z_\lambda)). \end{aligned} \tag{80}$$

This is the initial condition of the fluctuations in the open universe.

Next consider the discrete modes. We put $p^2 = -\Lambda^2$. The Bunch-Davis positive frequency mode is given by

$$\delta\phi^+_{\Lambda lm} = S_\Lambda(\sigma) Y_{\Lambda lm}(\tau,\Omega), \tag{81}$$

where

$$Y_{\Lambda lm}(\tau,\Omega) = \sqrt{\frac{\Gamma[\Lambda+l+1]\Gamma[-\Lambda+l+1]}{2}} \frac{P^{-l-1/2}_{\Lambda-1/2}(i\sinh\tau)}{\sqrt{i\cosh\tau}}, \tag{82}$$

and S_Λ is normalized as

$$\int_{-1}^1 dz |S_\Lambda(z)|^2 = 1. \tag{83}$$

From the regularity condition similar to the valley bounce, the solution is given by

$$\tilde{S}_\Lambda(z) = \begin{cases} \dfrac{\alpha_\Lambda}{\sqrt{1-z^2}}\Big(P^\Lambda_{\nu_F}(z) + \beta_\Lambda P^{-\Lambda}_{\nu_F}(z)\Big), & -1 \le z < z_\lambda, \\ \dfrac{1}{\sqrt{1-z^2}} P^{-\Lambda}_{\nu_T}(z), & z_\lambda \le z \le 1, \end{cases} \tag{84}$$

where

$$\beta_\Lambda = \frac{\sin\pi\nu_F}{\pi}\Gamma[1+\Lambda+\nu_F]\Gamma[\Lambda-\nu_F]. \tag{85}$$

From the junction condition, α_Λ is given by

$$\alpha_\Lambda = \left.\frac{P^{-\Lambda}_{\nu_T}}{P^\Lambda_{\nu_F} + \beta_\Lambda P^{-\Lambda}_{\nu_F}}\right|_{z_\lambda}, \quad \alpha_\Lambda = \left.\frac{d_z P^{-\Lambda}_{\nu_T}}{d_z P^\Lambda_{\nu_F} + \beta_\Lambda d_z P^{-\Lambda}_{\nu_F}}\right|_{z_\lambda}. \tag{86}$$

Then Λ is determined by the equation

$$P_{\nu_T}^{-\Lambda}(d_z P_{\nu_F}^{\Lambda} + \beta_\Lambda d_z P_{\nu_F}^{-\Lambda}) = d_z P_{\nu_T}^{-\Lambda}(P_{\nu_F}^{\Lambda} + \beta_\Lambda P_{\nu_F}^{-\Lambda}). \tag{87}$$

The mode $\tilde{S}_\Lambda$ is not normalized. The normalized mode is given by

$$S_\Lambda(z) = N_\Lambda\, \tilde{S}_\Lambda(z), \quad N_\Lambda = \left(\int_{-1}^{1} dz |\tilde{S}_\Lambda(z)|^2\right)^{-1/2}. \tag{88}$$

Inside the bubble, making the analytic continuation, we obtain the positive frequency mode

$$\delta\phi_\Lambda^+ = N_\Lambda\, \frac{P_{\nu_T}^{-\Lambda}(\cosh t)}{\sinh t} Y_{\Lambda lm}(r, \Omega). \tag{89}$$

In the limit $t \to 0$, this becomes

$$\delta\phi_\Lambda^+ = \frac{N_\Lambda}{2\Gamma[1+\Lambda]} T_\Lambda(\eta) Y_{\Lambda lm}(r, \Omega), \tag{90}$$

where $T_\Lambda = e^{\Lambda\eta - \eta}$. Using this mode function, the fluctuations can be expanded as

$$\delta\phi = \sum_i \sum_{lm} \delta\phi_{\Lambda_i}^+ \hat{\boldsymbol{a}}_{\Lambda_i} + (h.c). \tag{91}$$

In Fig.8 we plot the solutions Λ for $m_T^2 < 4$ (Case (2) in section 3.1). We also show the normalization factor N_Λ. We find two solutions of Λ. We call the mode with $0 < \Lambda_{sub} < 1$ the subcritical mode and the one with $1 < \Lambda_{sup}$ the supercritical mode [20]. In the case the background solution is given by the HM solution, one supercritical mode with $\Lambda_{sup} = \nu_T = \sqrt{9/4 + m_T^2} - 1/2 > 1$ exists. In the present case, the mass changes m_0^2 to $-m_T^2$ at z_λ, another subcritical mode appears. Note that in the case the CD solution describes the tunneling, the supercritical mode corresponds to the wall fluctuation mode $\delta\phi_w \propto \dot{\phi}_{CD}$ with $\Lambda_{sup} = 2$. Although in the present case the correspondence cannot be held, the behavior of the supercritical mode resembles the wall fluctuation mode.

5 An open universe from valley bounce

Using the results developed so far, we will study the model which gives an open universe in the bubble. Following, we restore the Hubble scale H. Since the radius of the bubble R is small compared with the Hubble horizon [9], then the curvature scale is greater than the energy of the matter inside the bubble ρ_M even if the whole energy of the false vacuum is converted to it, $\rho_M/M_p^2 \sim H^2 < 1/R^2$ [2]. Then, we need the second inflation in the bubble. To realize the second inflation inside the bubble, the field should roll slowly down the potential. This implies the curvature of the potential is small compared with the Hubble. To avoid the *ad hoc* fine-tuning of the potential, we will assume this is true for all region of the potential. In this case, since $m_T < H$ the solution of the Euclidean equation is given by the HM solution and the valley bounce is shown as in Fig.7. We connect the linear potential at the point the field appears after the tunneling $\phi = \phi_*$,

$$V(\phi) = V_* - \mu^3(\phi - \phi_*), \qquad (\phi > \phi_*). \tag{92}$$

We demand the potential and its derivative are connected smoothly at the connection point ϕ_*. Then we obtain

$$\begin{aligned} V_* &= \epsilon + \eta - \frac{1}{2} m_T^2 (\phi_* - \varphi_T)^2, \\ \mu^3 &= m_T^2 (\phi_* - \varphi_T). \end{aligned} \tag{93}$$

The initial conditions of the field are given by the valley bounce

$$\phi(t=0) = \phi_0(z=1) = \phi_*, \quad \dot{\phi}(t=0) = 0. \tag{94}$$

If the field obeys the classical field equation;

$$\ddot{\phi} + 3\coth t\, \dot{\phi} + V'(\phi) = 0, \tag{95}$$

then the solution of ϕ satisfies

$$\dot{\phi}(t) = \mu^3 \, \frac{\cosh^3 t - 3\cosh t + 2}{3\sinh^3 t}. \tag{96}$$

In the small t this behaves as $(1/4)\mu^3 t$. The classical motion during one expansion time is given by $|\dot{\phi}|H^{-1}$. On the other hand the amplitude of the quantum fluctuation is given by $\delta\phi \sim H$. The curvature perturbation $\mathcal{R}$ produced by the quantum fluctuation is approximately given by the ratio of these two quantities;

$$\mathcal{R} \sim \frac{\delta\phi}{|\dot{\phi}|H^{-1}} \sim \frac{H^3}{\mu^3} \sim \frac{H^2}{m_T^2}\left(\frac{H}{\phi_* - \varphi_T}\right). \tag{97}$$

This should be of the order 10^{-5} from the observation of the cosmic microwave background (CMB) anisotropies. If $|\phi_* - \varphi_T| < H$, as in the case the HM solution describes the tunneling, $\mathcal{R} > 1$ and the scenario cannot work well. This is because at $\phi_* \sim \varphi_T$, the field experiences the quantum diffusion rather than the classical potential force. Fluctuations in this diffusion dominated epoch make the inhomogeneous delay of the start of the classical motion, thus make large fluctuations. Fortunately, from Fig.7, we see for appropriate λ, the valley bounce gives the initial condition as $|\phi_* - \varphi_T| \sim O(1)(M^2/m_T)$, which is larger than the Hubble if $M > H$. In this case, the potential force works and the field rolls slowly down the potential according to eq.(95). We expect the curvature perturbation can be suppressed for the valley bounce

To confirm this expectation, consider the evolution of the initial fluctuation given in the last section. The scalar field fluctuations give rise to a metric perturbations. First consider the continuous modes [18]. The evolution equation for the gauge invariant gravitational potential Φ is given by

$$\Phi_p'' - \frac{6(1-e^{2\eta})}{3-2^{\eta}}\Phi_p' + \left(p^2 + 5 - \frac{4(3+e^{2\eta})}{3-e^{2\eta}}\right)\Phi_p = 0. \tag{98}$$

From this we see for small t, Φ_p behaves as $T_{\pm p}(\eta)e^{2\eta}$ and for general t, Φ_p behaves as

$$\Phi_p \sim T_{\pm p}(\eta)e^{2\eta}\left(1 - \frac{p \mp i}{3(p \pm i)}e^{2\eta}\right). \tag{99}$$

Furthermore for small t, this metric perturbation is related to the fluctuations of the scalar field by

$$\Phi_p \sim \frac{4\pi G\mu^3}{(\mp ip + 2)H^2}e^{2\eta}\ \delta\phi_p. \tag{100}$$

The initial fluctuations of the scalar field are given in eq.(79). Then, the metric perturbation generated during the second inflation is given by

$$\begin{aligned}
\Phi &= -i\frac{G\mu^3}{H}\sum_{lm}\int_0^\infty dp\frac{1}{2\sqrt{2p}}Y_{plm}(r,\Omega)\frac{1}{\sqrt{2\sinh\pi p}} \\
&\times \Big[\Big((e^{\pi p/2}g_1(p,\lambda)\tilde{T}_p(\eta) + e^{-\pi p/2}g_2(p,\lambda)\tilde{T}_{-p}(\eta))\hat{\boldsymbol{a}}_p \\
&+ (e^{-\pi p/2}g_1^*(p,\lambda)\tilde{T}_{-p}(\eta) + e^{\pi p/2}g_2^*(p,\lambda)\tilde{T}_p(\eta))\hat{\boldsymbol{a}}_{-p}\Big) + (h.c.)\Big],
\end{aligned} \tag{101}$$

where

$$\tilde{T}_{\pm p}(\eta) = T_{\pm p}(\eta)\frac{e^{2\eta}}{\mp ip + 2}\left(1 - \frac{p \mp i}{3(p \pm i)}e^{2\eta}\right). \tag{102}$$

The variable which has a normalization that relates more directly to the density perturbation after reheating is given by $\mathcal{R} = 16\pi G(V^2/V_{,\phi}^2)\Phi$. We define the power spectrum of $\mathcal{R}$ by

$${}_{BD}\langle 0|\mathcal{R}(r,\eta)\mathcal{R}(r',\eta)|0\rangle_{BD} = \sum_{lm}\int_0^\infty dp\ \ Y_{plm}(r)Y_{plm}(r')P_{\mathcal{R}}(p,\eta), \tag{103}$$

where $\hat{\boldsymbol{a}}_p|0\rangle_{BD} = 0$. Taking the limit $\eta \to 0 (t \to \infty)$, we obtain

$$\begin{aligned}
P_{\mathcal{R}}(p,\lambda) &= P_{BD}(p) \times \\
&\left[|g_1(p,\lambda)|^2 + |g_2(p,\lambda)|^2 - \frac{1}{\cosh\pi p}\left(\frac{p-i}{p+i}g_1(p,\lambda)g_2^*(p,\lambda) + \frac{p+i}{p-i}g_1^*(p,\lambda)g_2(p,\lambda)\right)\right],
\end{aligned} \tag{104}$$

where

$$P_{BD}(p) = \left(\frac{3H^3}{\mu^3}\right)^2 \frac{\coth \pi p}{2p(p^2+1)}. \tag{105}$$

The power of the continuous modes in the logarithmic interval p at $p \gg 1$ is given by

$$\lim_{p\to\infty} \frac{p^3}{2\pi^2} P_{\mathcal{R}}(p,\lambda) = \frac{1}{4\pi^2}\left(\frac{3H^3}{\mu^3}\right)^2 \sim \left(\frac{M_*^2}{M_p M}\right)^4 \left(\frac{H}{m_T}\right)^2. \tag{106}$$

Here we use the fact the valley bounce gives the initial condition as $|\phi_* - \varphi_T| \sim M^2/m_T$, then $\mu^3 = m_T M^2$. This quantity should be of the order 10^{-10} from the observation. This can be achieved by taking $(M_*^2/M) \ll M_p$. We show the dependence of λ in $P_{\mathcal{R}}$ in Fig.9.

The discrete modes can be treated in the same way. Φ_Λ generated from $\delta\phi_\Lambda$ is given by

$$\Phi_\Lambda = \sum_{lm} \frac{2\pi\mu^3 N_\Lambda}{H\Gamma[1+\Lambda]} \tilde{T}_\Lambda(\eta) Y_{\Lambda lm}(r_R, \Omega), \tag{107}$$

where

$$\tilde{T}_\Lambda = T_\Lambda(\eta)\frac{e^{2\eta}}{\Lambda+2}\left(1 + \frac{1-\Lambda}{3(1+\Lambda)}e^{2\eta}\right). \tag{108}$$

Taking the limit $\eta \to 0$, we obtain the power spectrum of $\mathcal{R}$

$$P_{\mathcal{R}}(\Lambda_i, \lambda) = \left(\frac{3H^3}{\mu^3}\right)^2 \left(\frac{N_{\Lambda_i}(\lambda)}{\Gamma[2+\Lambda_i(\lambda)]}\right)^2. \tag{109}$$

In some open inflation model, the contribution of the discrete modes gives the strong constraint on the model [22]. The supercurvature mode produces the very large scale metric perturbations and enhances the amplitude of the low multipoles of the CMB anisotropies. No evidence for such enhancement in the observed spectrum implies that the contribution of these discrete modes must not dominate the contribution of the continuous mode. Since the last factor in the power spectrum $P_{\mathcal{R}}(\Lambda_i, \lambda)$ is $O(1)$ (see Fig.8), the power of the discrete mode is comparable to that of the continuous modes. This result can be deduced from the analysis of the case the CD solution describes the tunneling and the thin-wall approximation can be used. In this case, the supercritical mode is the wall fluctuation mode given by $\delta\phi_w = N_w(\dot{\phi}_{CD})$. Here the normalization constant is given by $N_w = (\int d\sigma a(\sigma)\dot{\phi}_{CD}^2)^{-1/2}$. Within the thin wall approximation, this can be evaluated as $N_w \sim (RS_1)^{-1/2}$, where S_1 is the surface tension of the wall and R is the radius of the bubble. The surface tension of the wall is estimated by $S_1 \sim m_T(\triangle\phi)^2$, where $\triangle\phi$ is the scale of the variation of the field during tunneling. The curvature perturbation generated from this wall fluctuation is given by $\mathcal{R} \sim (H/\dot{\phi}_{CD})\delta\phi_w \sim H/\sqrt{RS_1}$. Using $R \sim 1/H$ and $m_T \sim H$, this can be estimated as $\mathcal{R} \sim H/\triangle\phi$. Thus, the thickness of the barrier the field passes during the tunneling determines the amplitude of the curvature perturbation generated from the wall fluctuation mode. In the case the valley bounce describes the tunneling, the supercritical mode can not be interpreted as the wall fluctuation mode. But the behavior of the supercritical mode resembles that of the wall fluctuation mode. Thus we expect this analysis can be applied. Since the valley bounce gives $\triangle\phi \sim M^2/m_T$, the contribution of the supercritical mode is suppressed. Then, the constraint from the discrete mode is not strong in this model.

6 Conclusion

It is difficult to provide the model which solves the horizon problem and at the same time leads to the open universe in the context of the usual inflationary scenario. In the one bubble open inflationary scenario, the horizon problem is solved by the first inflation and the second inflation creates the universe with the appropriate Ω_0.

Many works have been done within this framework of the scenario and it is recognized this scenario requires additional fine-tuning [3, 4]. The defect is thought to arise because the curvature around the barrier should be larger than the Hubble scale to avoid large fluctuations, which contradicts to the requirement that the curvature of the potential should be small to realize the second inflation inside the bubble. Additional constraint comes from the fluctuations generated in the decay process, which can be observed and reject some model [22].

Thus to complete the scenario, we should solve these problems. The main claim of this paper is that these problem can be solved in the simple model with the polynomial form of the potential. We reconsidered the tunneling process from the different perspective. If the curvature around the potential is small, the tunneling is described by one of a family of the almost saddle-point solutions [6]. This is because the true saddle-point solution, that is, the Hawking Moss solution does not satisfy the boundary condition for the false vacuum decay. The main idea is that the almost saddle-point solution can give the appropriate initial condition for the second inflation. A family of the almost saddle-point solutions generally forms a valley line in the functional space of the action, since these solutions have almost the same action with that of the Hawking Moss solution. We called the configurations on the valley line of the action valley bounces. To identify valley bounces, we applied the valley method developed by Aoyama.et.al [8]. In this method these configurations can be identified using the fact the trajectory they form in the functional space of the action corresponds to the line on which the action varies most gently. We formulated this method in the de Sitter spacetime and clarified the structure of valley bounces. We found the valley bounce which gives the appropriate initial condition of the second inflation even if the curvature around the barrier is small compared with the Hubble scale.

Consider the case this valley bounce describes the tunneling. It is possible the field appears sufficiently far from the top of the barrier after the tunneling, then we can avoid the large fluctuations. The fluctuations of the field give rise to the metric perturbation. These can be observed in our open universe. The fluctuations around the valley bounce which are orthogonal to the gradient of the action are relevant to the observable. We calculated the power spectrum of the metric perturbations generated in the second inflation and found these fluctuations can be made compatible with the observations. In some model of the open inflation, the discrete mode of the fluctuations gives strong constraint on the model. We showed this is not the case in our model. Hence, using the valley bounce, we can solve the problem which arises in the open inflationary scenario besides the usual fine-tuning of the inflationary scenario. The one bubble open inflation model can be constructed without difficulty.

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Figure captions

Fig.1 The piece-wise quadratic potential $V_T(\phi)$. Here, we take $m_T^2 = 7$, $m_F^2 = 2.2$ and $\eta = 0.6M^4$.

Fig.2 The behavior of the CD solution $\phi(\sigma)$, the eigenmode with the negative eigenvalue $g_-(\sigma)$ and the scale factor $a(\sigma)$. The potential is taken as in Fig.1.

Fig.3 The behavior of the valley bounce. The horizontal coordinate is σ. The lower-panel shows the behavior of the valley bounce on the trajectory associated with the negative eigenvalue and the upper-panel shows the behavior of the valley bounce on the trajectory associated with the smallest positive eigenvalue.

Fig.4 The action along the valley trajectory. The lower-panel shows the action of the trajectory associated with the negative eigenvalue. The horizontal coordinate is λ. The upper-panel shows the action of the trajectory associated with the smallest positive eigenvalue. The horizontal coordinate is the norm of the filed $\Phi = \sqrt{\int d\sigma a(\sigma)^3 |\phi(\sigma) - \varphi_T|^2}$.

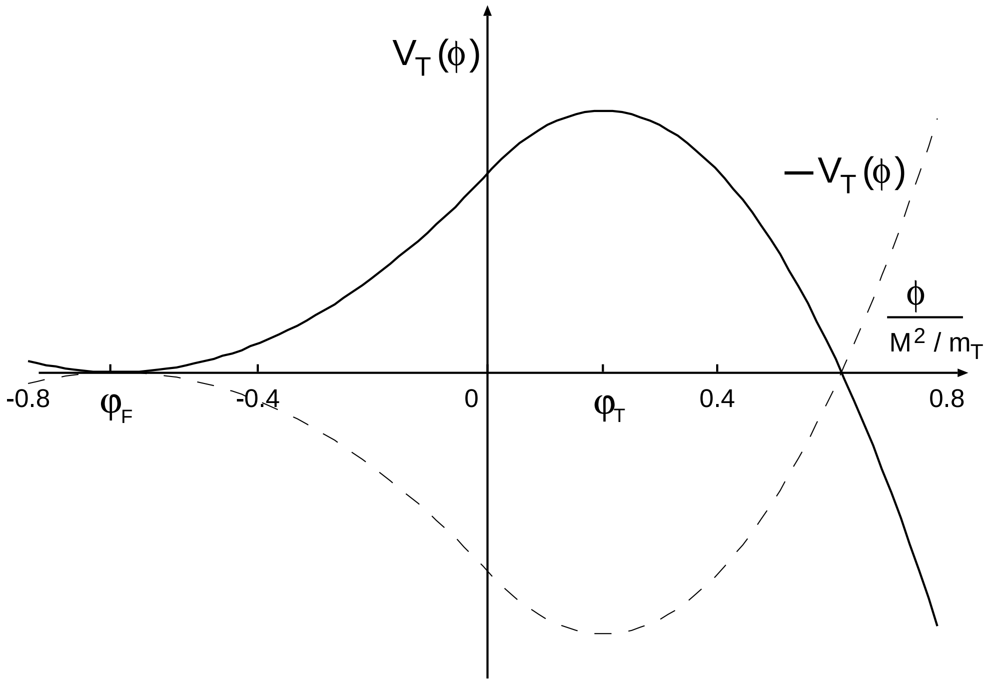
Fig.5 The piece-wise quadratic potential $V_T(\phi)$. We take $m_T^2 = 2$, $m_F^2 = 0.5$ and $\eta = 0.1M^4$.

Fig.6 The action along the valley trajectory associated with smallest positive mode. The potential is taken as in Fig.5.

Fig.7 The behavior of the valley bounce in the valley trajectory associated with lowest positive mode. The upper-panel shows the behavior of ϕ and the lower-panel shows the behavior of f. The potential is taken as in Fig.5.

Fig.8 The solution of Λ. Two solutions are shown. One corresponds to the subcritical mode $0 < \Lambda_{sub} < 1$ and another corresponds to the supercritical mode $1 < \Lambda_{sup}$. The corresponding valley bounces are shown in fig.7.

Fig.9 The power spectrum of the curvature perturbation around the Valley bounce (continuous mode). The corresponding valley bounces are shown in fig.7.



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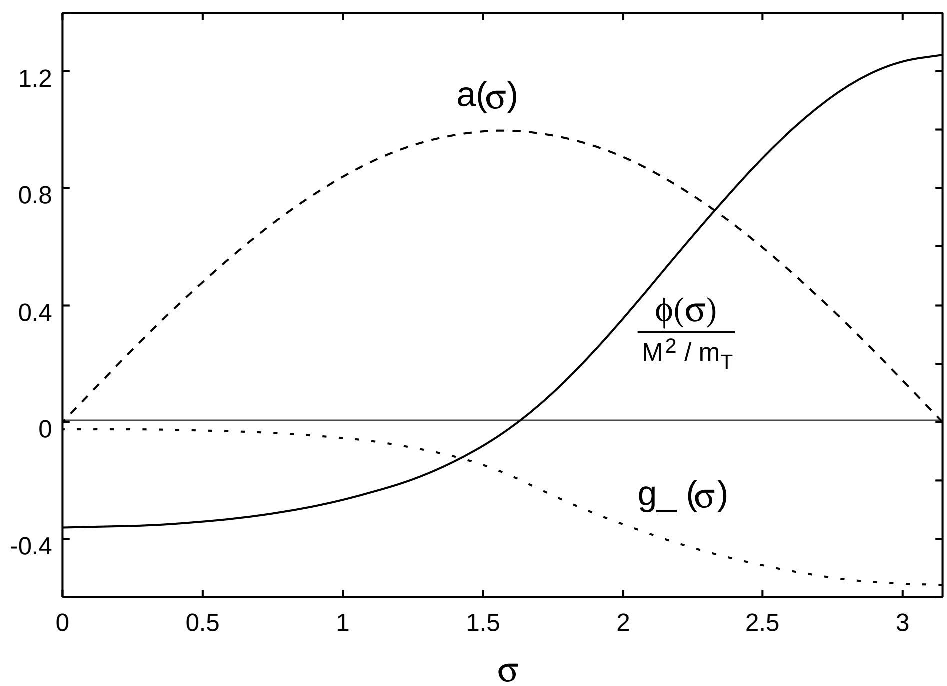

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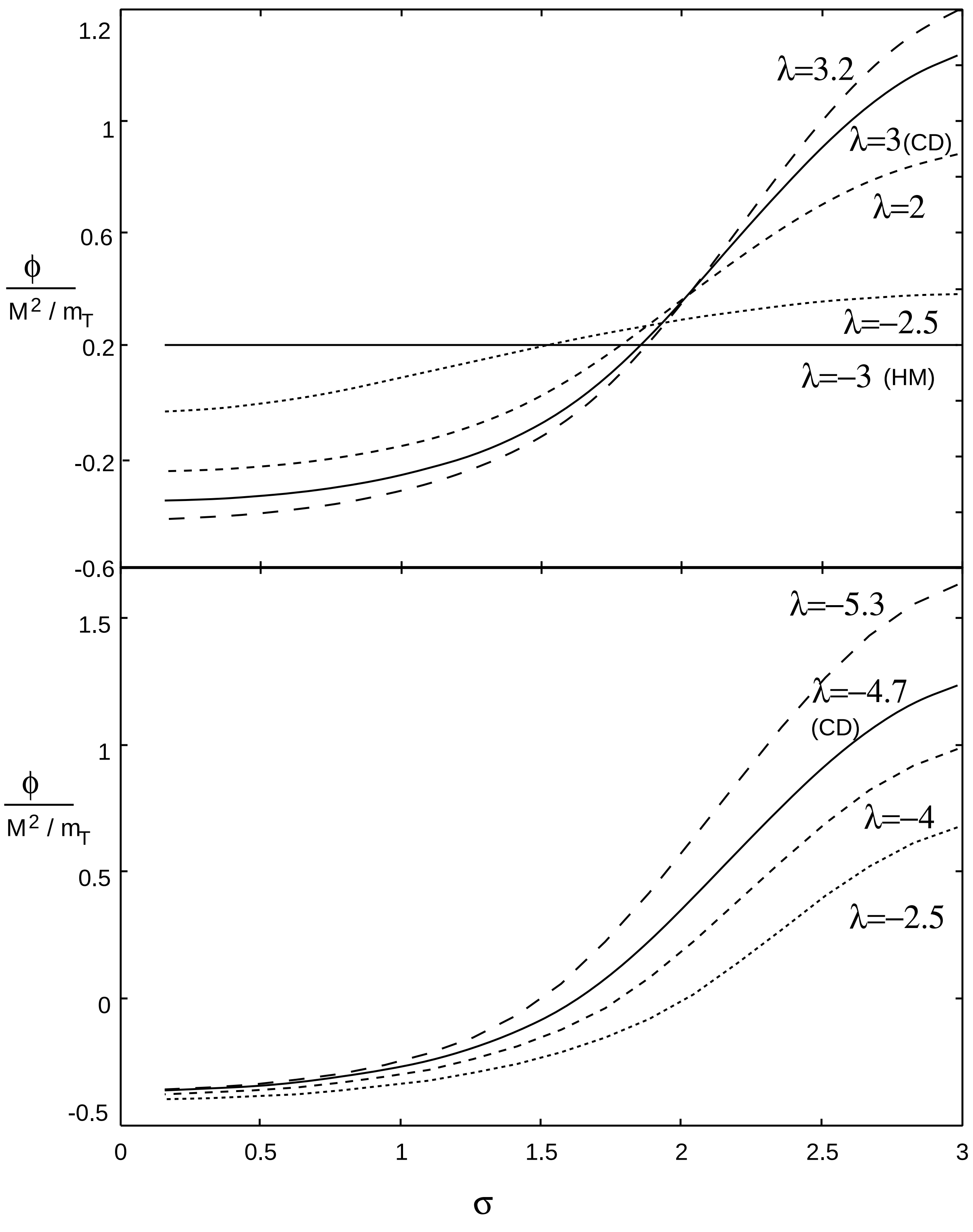

1.2
1
0.6
0.2
-0.2
-0.6
$\frac{\phi}{M^2 / m_T}$
λ=3.2
λ=3(CD)
λ=2
λ=−2.5
λ=−3 (HM)
1.5
1
0.5
0
-0.5
λ=−5.3
λ=−4.7
(CD)
λ=−4
λ=−2.5
0
0.5
1
1.5
2
2.5
3
σ

Figure 3:

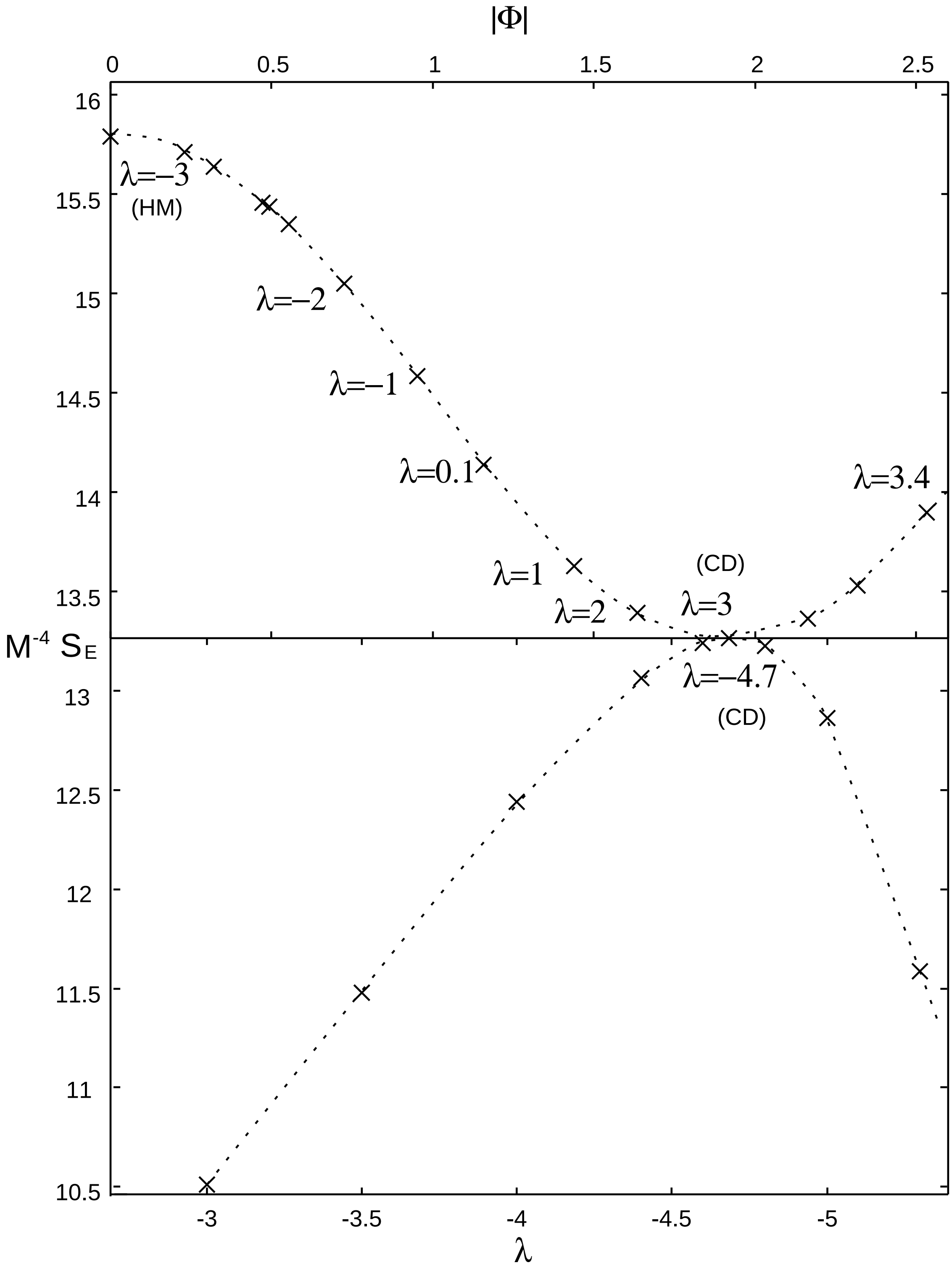
|Φ|
0
0.5
1
1.5
2
2.5
16
15.5
15
14.5
14
13.5
13
12.5
12
11.5
11
10.5
M-4 SE
λ=−3
(HM)
λ=−2
λ=−1
λ=0.1
λ=1
λ=2
λ=3
(CD)
λ=3.4
λ=−4.7
(CD)
-3
-3.5
-4
-4.5
-5
λ

Figure 4:

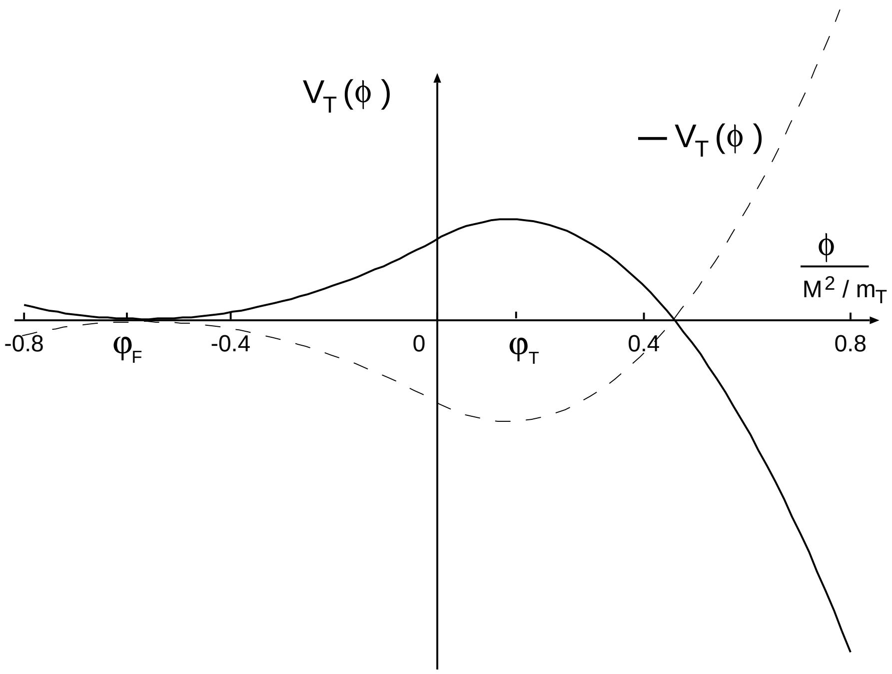

Figure 5:

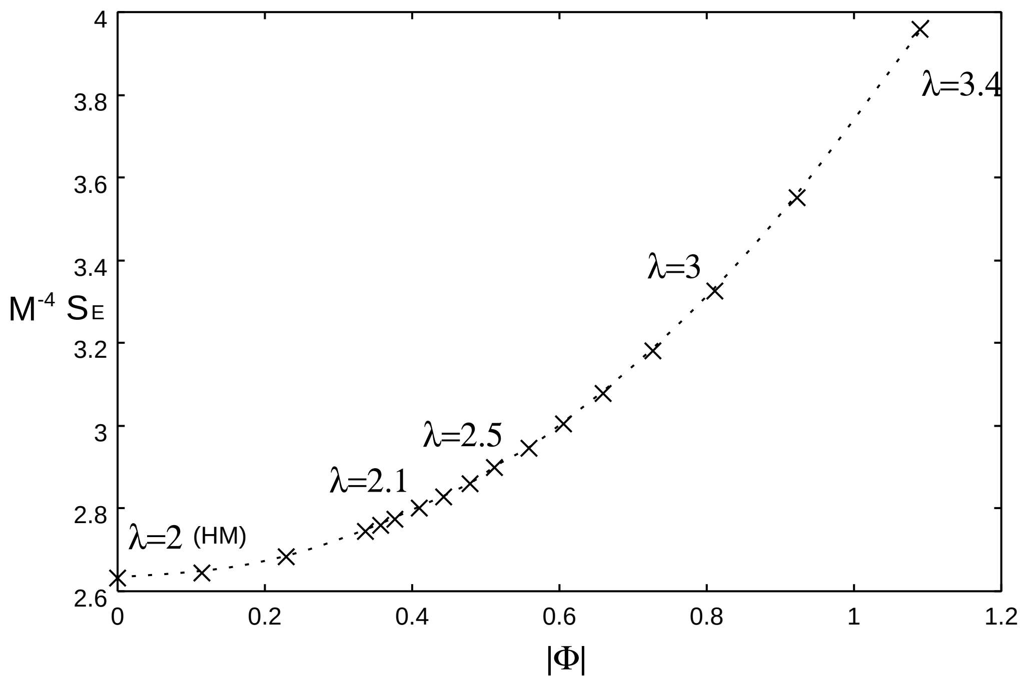

Figure 6:

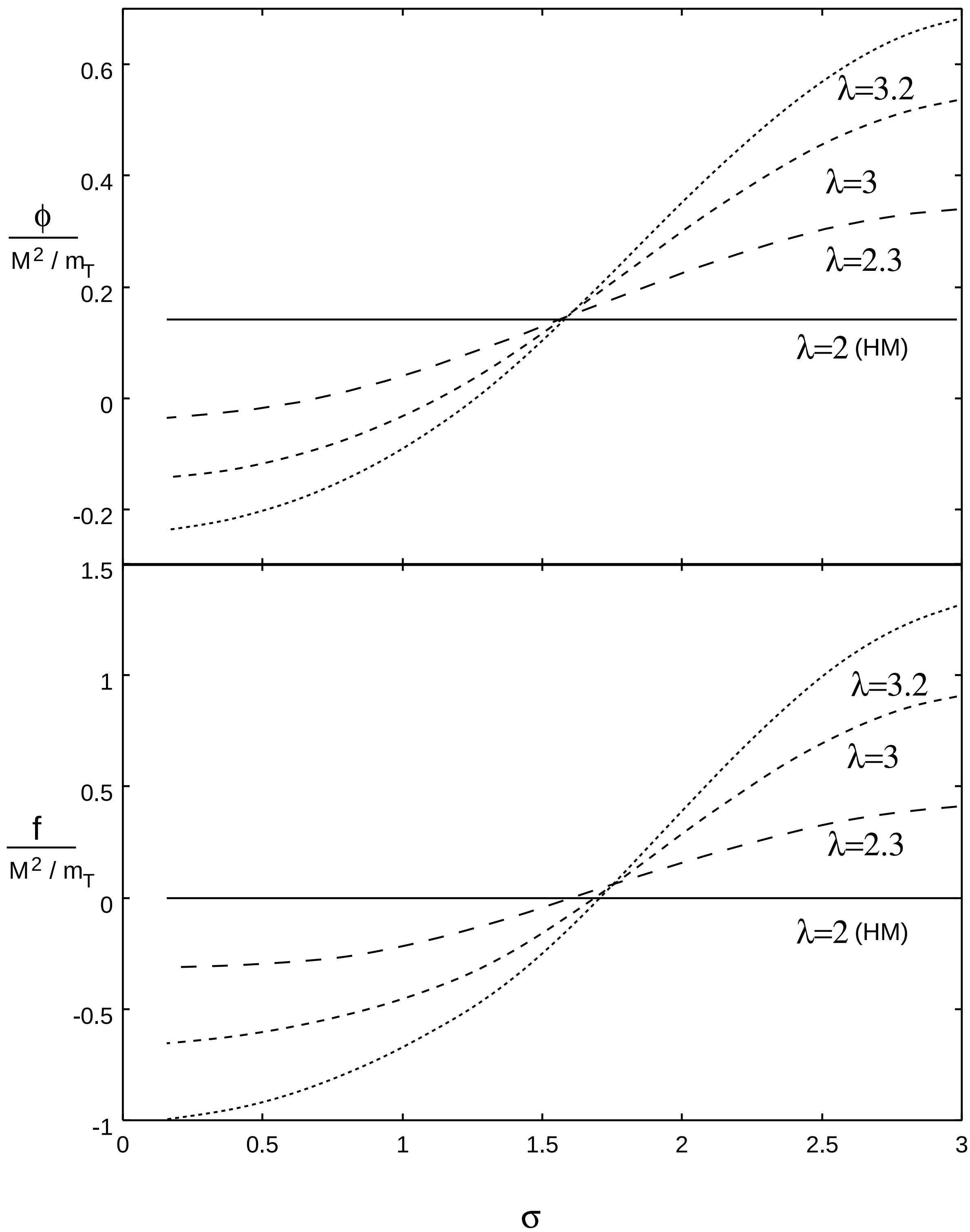
0.6
0.4
0.2
0
-0.2
$\frac{\phi}{M^2/m_T}$
λ=3.2
λ=3
λ=2.3
λ=2 (HM)
1.5
1
0.5
0
-0.5
-1
$\frac{f}{M^2/m_T}$
λ=3.2
λ=3
λ=2.3
λ=2 (HM)
0
0.5
1
1.5
2
2.5
3
σ

Figure 7:

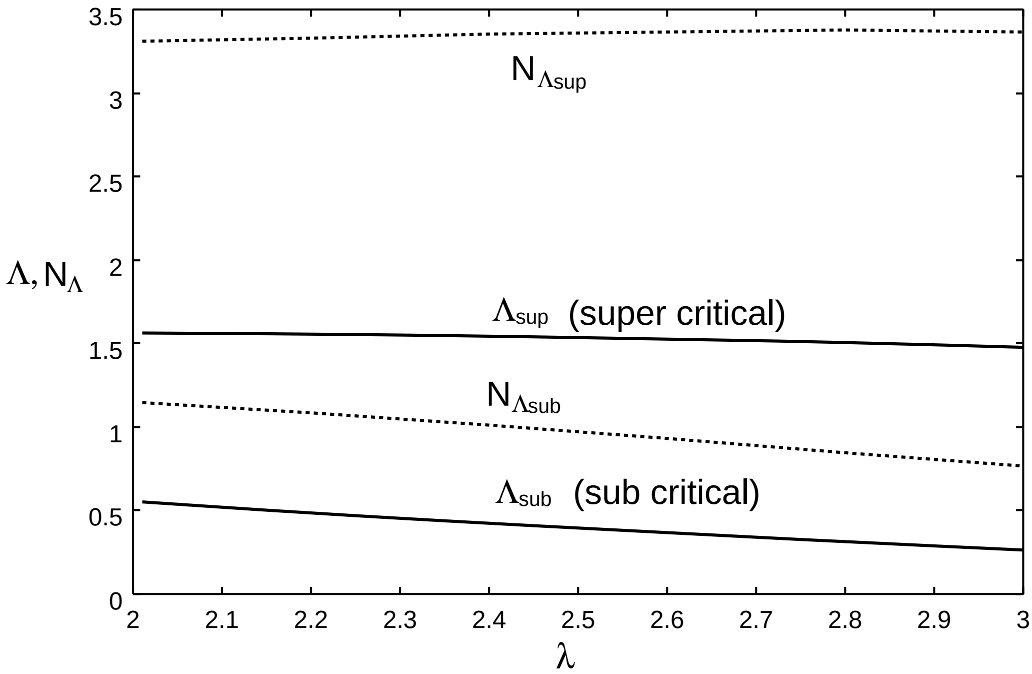

Figure 8:

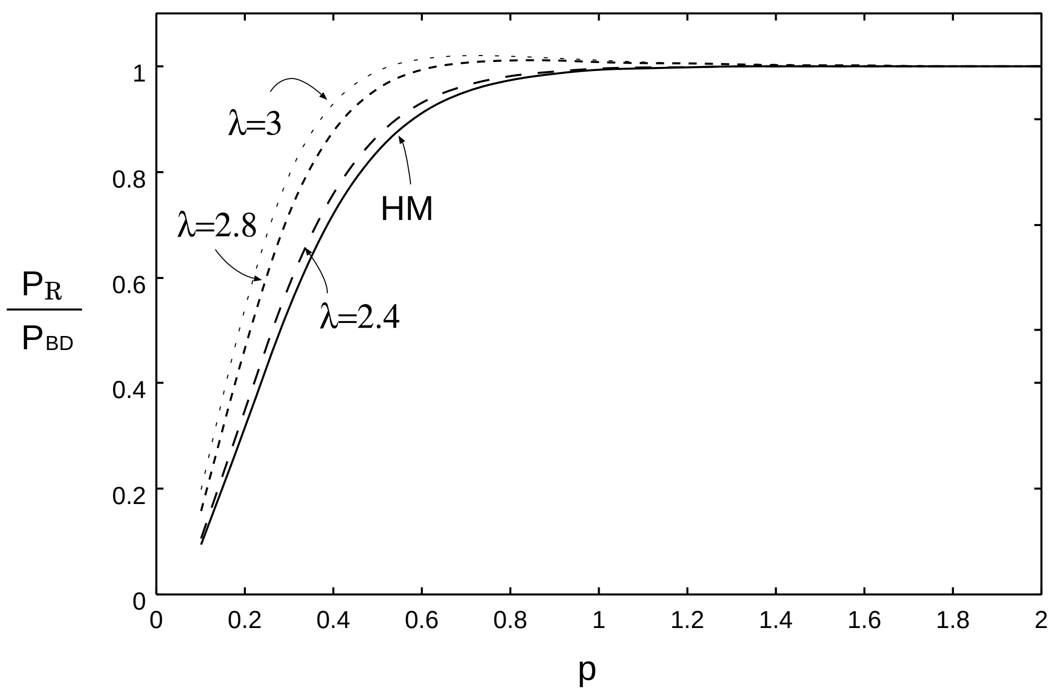

Figure 9: